Theorem (Squeezed sequence lemma)**.** *Let (a_n), (b_n) and (c_n) be three sequences of real numbers. Assume that*

(1) For all $n \in \mathbb{N}$, $a_n \leq b_n \leq c_n$.

(2) $\lim_{n \to \infty} a_n = \lim_{n \to \infty} c_n = a$.

Then $\lim_{n \to \infty} b_n = a$.

Proof. Fix $\varepsilon > 0$. Observe that if $a_n \in V_\varepsilon(a)$ and $c_n \in V_\varepsilon(a)$, then $b_n \in V_\varepsilon(a)$. Thus we have

$$\{n \, : \, b_n \notin V_\varepsilon(a)\} \subset \{n \, : \, a_n \notin V_\varepsilon(a)\} \bigcup \{n \, : \, c_n \notin V_\varepsilon(a)\} \, .$$

Since both sets in the right hand side are finite, so is the set in the left hand side. $\qquad \square$